The Calculation of Heat Transfer Coefficient for Combustion Driven Transverse Oscillations in a Gas-Air Burner

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In this paper a method is presented for the computation of the heat transfer coefficient in a gas-air burner in the presence of combustion-driven transverse mode oscillations. By considering the oscillations in pressure within the chamber to be small compared with the mean chamber pressure, and to be known, methods of acoustics may be used to determine the average energy density of the acoustic waves at the wall. Consideration of this acoustic energy to create greater mixing in the boundary layer enables the heat transfer problem to be considered by the Dankwerts-Mickley model for turbulent heat exchange. With suitable definition of a friction velocity in terms of the friction velocity v^* for fully developed turbulent pipe flow and the root mean square particle velocity of the acoustic wave ($a_{r.m.s.\omega}$), it is found that agreement exists between the computed ratio of heat transfer coefficients, with and without combustion-driven oscillations and the experiments of Zartman, which appear to be the only experimental data for transverse oscillations in a gas-air burner.

The occurrence of oscillations in combustion systems under certain conditions of operation was observed as early as 1777 (*1*), but with the advent of jet and rocket propulsion systems a systematic study of this phenomenon has been undertaken.

The types of oscillation to be considered here are those associated with the aerothermochemistry and acoustics of the combustion system, in which the frequency of the oscillation corresponds to one or more of the resonant acoustic modes pertinent to the geometry of the combustion chamber.

The acoustic modes in their pure form for a cylindrical geometry are longitudinal, transverse, and radial. The transverse mode can be either of a *Sloshing* or a *Spinning* form.

The above designation describes the motion of the fluid in the chamber. Figure 1 illustrates the various forms of oscillation in the absence of bulk flow of gas through the chamber.

In rocket motors (*2, 7*), where high-energy fuels are used at high combustion chamber pressures, combustion-driven oscillations are possible. These oscillations result in large fluctuations of pressure and velocity and increased heat transfer to the chamber walls, which can destroy the chamber in a very short period of time. However, Reynst (*3*), Francis et al. (*4*), and others (*1*), working with commercial hydrocarbons and pulverized solid fuels and air, at moderate combustion chamber pressures, have endeavored to investigate controlled combustion-driven oscillations for their application in reducing the size of industrial boiler plant.

The theoretical and experimental work reported in the unclassified literature (*1*, 5 to 8) has been associated in the main with rocket, jet afterburners, and ramjet systems, where the main concern has been the suppression of combustion-driven oscillations of all types. Therefore the aim of much of this work has been in determining the chemical, geometric, and aerodynamic factors giving rise to combustion oscillation, and to set up suitable limits within which stable operation of the system is possible. Since the transverse modes of oscillation have been found to give the largest increases in heat transfer, it is this mode which will receive attention here.

Any attempt at calculating the convective heat transfer coefficient between the combustion gases and the chamber wall by a consideration of the detailed fluid motion within the chamber would be extremely difficult. Therefore a semiempirical approach to the problem would appear more probable.

The frequency of fluctuations in pressure and velocity during combustion-driven oscillations can be readily calculated from acoustic theory, and the results have been found to be in good agreement with experiment (2, 5, 10). However, the magnitude of the pressure and velocity fluctuations cannot be predicted, because in the case of the rocket motor; the wave phenomenon caused by the coupling of the combustion heat release and the acoustic waves in the chamber are outside the scope of acoustics, which is based on the assumption of small perturbations. In rocket motors the transverse mode is usually of the spinning variety, whereas for the lower combustion intensity afterburner and ramjet the mode is likely to be of the sloshing type. One can therefore think of the spinning mode as an overdriven sloshing mode.

In the combustion chamber studied by Zartman (*10*), using a propane-air mixture at moderate combustion chamber pressure, the level of the combustion-driven oscillations was such that acoustic theory would be applicable. Before the magnitude of the pressure fluctuations in the combustion chamber can be calculated by acoustic methods, the phasing of heat release to the acoustic disturbances in the chamber due to flow and geometric considerations must be known. This type of information is not generally available and can only be obtained from experimental data. Zartman (*10*) did not obtain data of these types, but measured the pressure fluctuations at the wall. With the assumption, then, that from semiempirical data the fluctuations in pressure at the chamber wall are known for a given geometry, fuel, and flow condition, this paper presents a method by which the computation

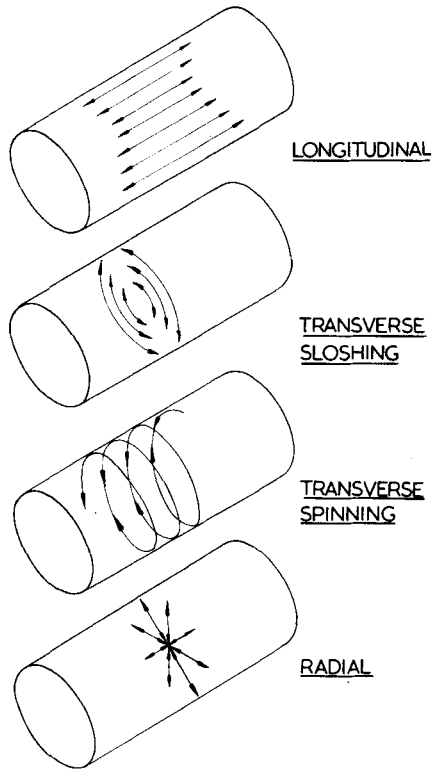

Fig. 1. Acoustic modes of oscillation for a cylindrical chamber.

of the convective heat transfer coefficient from the hot gases to the chamber wall may be carried out. The results obtained by the method of this paper are compared with the experimental data of Zartman (*10*), who was only concerned with the measurement of heat transfer coefficients during combustion-driven oscillations.

THEORY

In the study of interphase mass transfer, Dankwerts (*11, 12*) proposed the penetration model for the process. This model was later incorporated into a more general discussion by Toor and Marchello (*13*), who applied it to the case of heat or mass tranfser in turbulent pipe flow. In essence the heat exchange model of reference (*13*) considers the laminar sublayer adjacent to the pipe wall to be penetrated by *lumps* of fluid from the turbulent core and to exchange heat with the wall in a transient manner before being displaced into the core by fresh fluid lumps from that region, the whole process taking place randomly and at high frequency. A similar model was proposed by Mickley and Fairbanks (*14*) for the transfer of heat in fluidized beds. Here aggregates of particles are considered to move randomly to the containing wall, to exchange heat with it in a transient manner, and then to be replaced by fresh aggregates from the core of the bed. These models have been proposed for complex mixing processes because detailed mathematical description of the fluid motion is not possible.

From a photographic study of the spinning mode of transverse oscillations in a rocket motor, Male et al. (*2*) conclude that it is possible that the high rates of heat transfer were due to turbulence, which caused macroscopic transfer of the gas to the chamber wall. Similar gas motion was observed in the photographic studies of Krieg (*15*) on a rocket motor in which the spinning transverse mode was present. It is reasonable to assume that a similar gas motion would be observed with the sloshing transverse mode of gas oscillations, but with the absence of the travelling detonation front, which is present in high intensity combustion rocket systems which are in a spinning transverse mode of oscillation.

If one considers the effect of combustion-driven oscillations in the transverse sloshing mode to promote high-frequency mixing in the laminar sublayer at the wall, then it is possible to apply the Dankwerts-Mickley model to the calculation of the heat transfer coefficient between the gas and chamber wall.

From the Dankwerts-Mickley model the heat transfer coefficient between the fluid medium and the containing wall is given by

$$h = \sqrt{k \rho C_p S} \tag{1}$$

The details of the derivation of Equation (1) are to be found in references *13* and *14*. The important quantity in Equation (1) is the mixing coefficient S, which is controlled by the fluid dynamics and geometry of the particular situation. Toor and Marchello (*13*) show that for turbulent pipe flow S_o is proportional to

$$\frac{\overline{V}}{D} N_{Re}^{0.8}$$

In the theory of turbulent boundary layers (*16*) the friction velocity v^* is used as a measure of turbulent eddying, and of the transfer of momentum due to these eddies, and it is known (*16*) that v^* is proportional to $\overline{V}$. Therefore if S_o is set proportional to $v^*/D\ N_{Re}^{0.8}$, then

$$h_o = \sqrt{k \rho C_p \beta \frac{v^*}{D} N_{Re}^{0.8}} \tag{2}$$

It is shown in the Appendix that the value of h_o computed from Equation (2) with the selection of a suitable value of the proportionality constant β, will be in substantial agreement with the value of h_o computed from the McAdam correlation (*17*): $N_{Nu.} = 0.023\ N_{Re}^{0.8}\ N_{Pr}^{0.4}$ with $N_{Pr} = 0.73$ and $10{,}000 \leq N_{Re} \leq 120{,}000$.

The next step is to set up a form for the mixing coefficient S_v when a turbulent bulk flow with simultaneous transverse acoustic gas oscillations exists, Now

$$\frac{\rho v^{*2}}{g_c} = \tau_{wo} \tag{3}$$

Due to the gas oscillation, the average energy density (*18*) in the acoustic wave is

$$\frac{I}{C} = \frac{\rho u^2_{\text{max.}}}{2g_c} = \frac{\rho u^2_{\text{r.m.s.}}}{g_c}$$

for a wave of sinusoidal form

$$= \frac{\rho}{g_c} [a_{\text{r.m.s.}}\ \omega]^2 \tag{4}$$

If one considers that because of acoustic oscillations which are superimposed on the bulk flow, the average energy density of the acoustic wave at the wall supplements the frictional energy $\rho v^{*2}/g_c$, then the wall shearing stress τ_{wo} is modified to τ_{wv} where

$$\tau_{wv} = \frac{\rho}{g_c} [v^{*2} + (a_{\text{r.m.s.}}\ \omega)^2] \tag{5}$$

If a friction velocity v^{**} is defined, and the mixing coefficient S_v is set proportional to

$$\frac{v^{**} N_{Re}^{0.8}}{D}$$

where

$$v^{**} = \sqrt{v^{*2} + (a_{\text{r.m.s.}}\ \omega)^2}$$

then

$$h_v = \sqrt{k \rho C_p \beta \frac{v^{**} N_{Re}^{0.8}}{D}} \tag{6}$$

The proportionality constant must remain as β; if this were not so, then with

$$(a_{\text{r.m.s. }\omega}) = 0; \quad \frac{h_v}{h_o} \neq 1$$

$$\therefore \frac{h_v}{h_o} = \sqrt{\frac{v^{**}}{v^*}} = \left[1 + \left(\frac{a_{\text{r.m.s. }\omega}}{v^*}\right)^2\right]^{1/4} \qquad (7)$$

COMPARISON WITH EXPERIMENTAL DATA

For fully developed pipe flow (*16*)

$$\tau_{wo} = \frac{0.0333\,\rho\,\overline{V}^{7/4}}{g_c}\left[\frac{\nu}{R}\right]^{1/4}$$

Substituting for τ_{wo} in Equation (3) and rearranging, one gets

$$v^* = (0.0333)^{1/2}\,\overline{V}^{1/8}\left[\frac{\nu}{R}\right]^{1/8} \qquad (8)$$

where ν in Equation (8) is evaluated on a molal average basis at the mean of the gas and wall temperatures.

In the evaluation of $(a_{\text{r.m.s. }\omega})$ from the measured acoustic intensity, the following assumption was made. The measured sound pressure level at a point just upstream of the flameholder is the same as that at the point at which heat transfer measurements were made. This assumption appears reasonable in the light of experimental evidence (*18*) on a burner with flameholder in which a small peak pressure was observed just downstream of the flameholder; but over the remainder of the downstream length and part of the length upstream from the flameholder the pressure amplitude level was approximately constant. The following conversions were used in determining $(a_{\text{r.m.s. }\omega})$ from the measured sound intensity in decibels (*20*):

Sound pressure level (S.P.L.) decibels $= 20 \log_{10} \left[\frac{P}{P_o}\right]_{\text{r.m.s.}}$ where P_o = reference pressure level = 0.0002 dynes/sq.cm.

$$(a_{\text{r.m.s. }\omega}) = \frac{P_{\text{r.m.s.}}\, g_c}{\rho\, C}$$

Only transverse oscillation data for the 17½-in. burning length were taken from reference *10*, all property values being evaluated on an average molal basis at the average gas and wall temperature. The reasons for using the 17½-in. data were the effects of the combustion reaction at the point at which the heat transfer measurements were being made would be minimal, and the flow at that point corresponded closely to the fully developed condition on which the method presented in this paper is based.

Equation (7) is plotted in Figure 2, with the experimental data of reference *10* shown. A least squares plot of the data with a slope of ¼ assumed for a linear law is also shown. The standard deviation between the experimental data points and Equation (7) is 0.11.

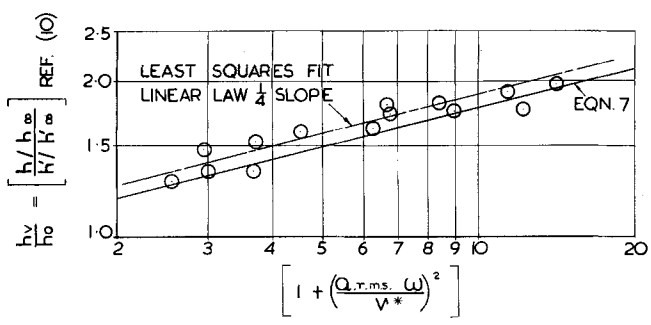

Fig. 2. Effect of acoustic oscillation on forced convective heat transfer.

A method has been presented by which the heat transfer coefficient between the gases and the wall of a combustion chamber may be computed, when the gases are in a transverse mode of oscillation in which the pressure fluctuations are small compared with the mean pressure in the chamber, and provided that the pressure fluctuations at the wall are known. The method indicates a continuous change in the heat transfer coefficient with intensity of oscillation, whereas Zartman observed increases in heat transfer occurred, only when the S.P.L. exceeded 130 decibels. However, at 130 decibels the increase predicted by the method here would fall within the range of normal experimental error for heat transfer during oscillation free combustion. At the high intensities, greater than 140 decibels, agreement exists between the method presented here and experiment. However, further experiments are required to verify completely the method put forward here. Experiments are also required to attempt to obtain a relation between burner geometry, operating conditions and air-fuel ratio, with S.P.L. within the chamber. If this can be done then all the information necessary for predicting the performance of complete burners employing combustion-driven transverse oscillations will be available.

NOTATION

a = particle amplitude, L
C = sonic velocity, L/t
C_1 = constant, dimensionless
C_p = constant-pressure thermal capacity L^2/t^2T
D = combustion chamber inner diameter, L
f = frequency of oscillation, $1/t$
g_c = gravitational constant, dimensionless
h = convective heat transfer coefficient, M/t^3T
I = intensity of acoustic wave M/Lt^2
k = thermal conductivity, ML/t^3T
N_{Nu} = Nusselt number, dimensionless
N_{Re} = Reynolds number, dimensionless
N_{Pr} = Prandtl number, dimensionless
P = pressure, M/t^2L
R = inner radius of combustion chamber, L
S = mixing coefficient, $1/t$
u = particle velocity, L/t
v^*; v^{**} = friction velocities, L/t
$\overline{V}$ = mean free stream velocity, L/t

Greek Letters

β = constant, dimensionless
ν = kinematic viscosity, L^2/t
ρ = density, M/L^3
τ_w = shear stress at wall, M/Lt^2
ω = circular frequency $= 2\pi f$ $(1/t)$

Subscripts

o = conditions of fully developed pipe flow and no oscillations
v = conditions of fully developed pipe flow with oscillations present

LITERATURE CITED

1. Markstein, G. H., ed., "Non Steady Flame Propagation," Pergamon Press, London (1964).
2. Male, T., W. R. Kerslake, and A. O. Tischler, *Natl. Advisory Comm. Aeronaut. Res. Memo E 54A 29* (1954).
3. Thring, M. W., ed., "Pulsating Combustion—Collected Works of E. H. Reynst." Pergamon Press, London (1961).
4. Francis, W. E., M. L. Hoggarth, and D. Reay, *Inst. Gas Eng. J.*, 3, No. 6, 301 (June, 1963).

5. Osborn, J. R., *Raketentech. Raumfahrtforschung*, 47 (Apr.-June, 1963).
6. ———, and J. M. Bonnell, *ARS J.*, 31, No. 4, 482 (Apr., 1961).
7. ———, *Jet Propulsion Center Rept. No.* 1-60-1, Purdue Univ., Lafayette, Ind. (1960).
8. Crocco, L., and S. I. Cheng, "Theory of Combustion Instability in Liquid Propellant Rocket Motors," Butterworths, London (1956).
9. Zartman, W. N., and S. W. Churchill, *AIChE J.*, 7, 588 (1961).
10. Zartman, W. N., Ph.D. thesis, Univ. Michigan, Ann Arbor (1960).
11. Dankwerts, P. V., *Ind. Eng. Chem.*, 43, 1460 (1951).
12. ———, *AIChE J.*, 1, 456 (1955).
13. Toor, H. L., and J. M. Marchello, *ibid.*, 4, 97 (1958).
14. Mickley, H. S., and D. F. Fairbanks, *ibid.*, 1, 374 (1955).
15. Krieg, Jr., H. C., paper presented at ARS Propellants Combustion and Liquid Rockets Conf., Florida (1961).
16. Schlichting, H., "Boundary Layer Theory," Pergamon Press, London (1955).
17. McAdams, W. H., "Heat Transmission," 3 ed., McGraw-Hill, New York (1954).
18. Ingard, U., and W. L. Kraushaar, "Introduction to Mechanics, Matter and Waves," Addison Wesley, Reading, Mass (1960).
19. *Natl. Advisory Comm. Aeronaut. Rept.* 1384 (1954).
20. Morse, P. M., "Vibration and Sound," McGraw-Hill, New York (1948).

APPENDIX

$$h_o = \sqrt{k_\rho C_p S_o}$$

For fully developed turbulent flow in a pipe

S_o proportional to $\dfrac{v^* N_{Re}^{0.8}}{D}$

$$\therefore h_o = \sqrt{k_\rho C_p \left[\frac{\beta v^* N_{Re}^{0.8}}{D}\right]}$$

where β = constant of proportionality

$$h_o \sqrt{\frac{D}{k_\rho C_p}} = \sqrt{\beta}\ \sqrt{v^*}\ N_{Re}^{0.4}$$

For fully developed turbulent pipe flow from reference *16*

$$v^{*2} = \frac{0.0396\ \overline{V}^{7/4}\, \nu^{1/4}}{D^{1/4}}$$

$$\therefore h_o \sqrt{\frac{D}{k_\rho C_p}} = C_1 N_{Re}^{0.4} \frac{\overline{V}^{1/2}}{N_{Re}^{1/16}}$$

where

$$C_1 = \sqrt{\beta}\,(0.0396)^{1/4}$$

$$\therefore \frac{h_o D}{k} = C_1 N_{Re}^{0.838} N_{Pr}^{0.5} \qquad (A1)$$

McAdam correlation reference *17*

$$\frac{h_o D}{k} = 0.023\, N_{Re}^{0.8} N_{Pr}^{0.4} \qquad (A2)$$

valid in range $10{,}000 \leq N_{Re} \leq 120{,}000$

$$0.7 \leq N_{Pr} \leq 120$$

The results from (A1) and (A2) agree to within 6% when $C_1 = 0.0162$ and $N_{Pr} = 0.73$ for the range $10{,}000 \leq N_{Re} \leq 120{,}000$; a slight divergence in results occurs with increasing N_{Re}.

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Permeation of Vapors Through Polymers at Low Temperature and Elevated Pressures

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The permeation of propane, propylene, ethane, and ethylene through 0.0051-cm. thick polyethylene films was measured at temperatures between 25° and −30°C. Plots of P vs. $1/T$ for C_3H_8, C_3H_6, and C_2H_6 were saucer shaped, the activation energies changing from positive (endothermic) to negative (exothermic) over a narrow temperature range. This change is attributed to a plasticization-condensation phenomenon and a change in intermolecular potentials.

Membrane permeation is a diffusional process which, to a first approximation, follows Fick's law:

$$N = -D \frac{dC}{dz} \qquad (1)$$

If D is independent of concentration, Equation (1) can be integrated with ($z = 0$, $C = C_1$, $z = z$, $C = C_2$) to

$$N = \frac{D}{z}(C_1 - C_2) \qquad (2)$$

Assuming the solubility constant S is only a function of temperature and not pressure and $C = Sp$, we have

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